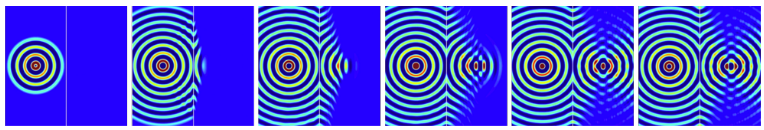
Waves in periodic media and metamaterials
Scientific Program

November 23rd - 25th, Cargese



Wednesday, November 23rd

9:00 am - 9:45am. *Assyr Abdulle (EPFL, Lausanne) and various collaborators* - **Two stories on wave approximation in heterogenenous media.**

Numerical approximation of wave equations in heterogenenous media is challenging as standard numerical methods require grid resolution down to the finest scale for convergence. In this talk we review recent works on the construction and analysis of efficient multiscale schemes for such problems. For problems without scale separation we show that discrete coarse approximation spaces based on local orthogonal decomposition can be used to capture the wave dynamics. For periodic problems, we discuss analytical and numerical model that are able to capture the dispersive effects that appear when the wave propagates over long time.

This talk is based upon a series of joint works with various collaborators (see the references below).

(1) A. Abdulle, M. Grote and C. Stohrer, Finite element heterogeneous multiscale method for the wave equation: long-time effects, SIAM MMS 2014.
(2) A. Abdulle and P. Henning, Localized orthogonal decomposition method for the wave equation with a continuum of scales, Math. Comp. 2016.
(3) A. Abdulle and T. Pouchon, A priori error analysis of the finite element heterogenenous multiscale method for the wave equation over long time, SIAM J. Numer. Anal, 2016.
(4) A. Abdulle and T. Pouchon, Effective models for the multidimensional wave equation in heterogeneous media over long time, to appear in Math. Models Methods Appl. Sci., 2016

9:45 am - 10:30am. *Eliane Bécache (POEMS, Inria, Université Paris Saclay), Maryna Kachanovska (POEMS, ENSTA, Université Paris Saclay)* - **Stable Perfectly Matched Layers for a Class of Anisotropic Dispersive Models.**

We consider a problem of a computational modelling of the two-dimensional wave propagation in dispersive anisotropic media in an infinite or semi-infinite domain. In particular, we concentrate on a simplified case of two-dimensional Maxwell equations with a diagonal anisotropic tensor of dielectric permittivity and scalar magnetic permeability, where all the quantities depend on the frequency nontrivially and correspond to passive materials.

To bound the computational domain, we suggest to use the perfectly matched layer (PML) technique. However, when applied to the problems under consideration, classical perfectly matched layers exhibit instabilities connected to the presence of so-called backward propagating waves. We will show how the PMLs can be stabilized for the problem under consideration. If time permits, we will discuss necessary and sufficient conditions of stability of PMLs for a particular case of nondissipative Lorentz materials, as well as show how one can derive energy estimates for the resulting PML system.

10:30 am - 11:00am. Coffee Break.

11:00 am - 11:45am. *Didier Felbacq* *(Laboratoire Charles Coulomb, Université de Montpellier)* - **Strong and weak coupling of a quantum emitter with a metasurface: a game of poles.**

Metasurfaces and the two dimensional analogue of metamaterials. They consist of resonant elements disposed on the surface. They are generally considered in the context of light propagation control, for instance as a mean to control polarization or to obtain generalized refraction laws.

In the present work, we are interested in the electromagnetic surface modes that can exist on these metasurfaces. These Bloch modes arise due to the presence of resonances and form flat bands. These modes shaw interesting features by themselves:they lead to rapidly varying properties of the metasurfaces with respect to the wavelength

We used these modes in order to reach various coupling regimes between the metasurface and a quantum emitter, modelized semi-classically by an oscillator. Using multiple scattering theory and complex plane techniques, we show that the coupling can be characterized by means of a pole-and-zero structure. The regime of strong coupling is shown to be reached when the pole-and-zero pair is broken.

11:45 am - 12:30am. *Ben Schweizer* *(TU Dormund) and Agnes Lamacz (TU Dormund)* - **Outgoing Wave Conditions in Photonic Crystals and Transmission Properties at Interfaces.**

We investigate the following transmission problem: a wave travels in free space and hits the boundary of a photonic crystal. Waves are described with Helmholtz equations, in the photonic crystal the Helmholtz equation has x-dependent coefficients. In experiments, a multitude of effects can be observed: (1) perfect reflection. (2) (partial) transmission with (a) positive refraction and (b) negative refraction. (3) creation of localized interface waves. An important step in the analysis of the problem regards the outgoing wave condition in the photonic crystal. The Sommerfeld condition, which is only applicable in free space, must be replaced by an adequate radiation condition. We develop an outgoing wave condition with the help of a Bloch wave expansion. Our radiation condition admits a (weak) uniqueness result which is formulated in terms of the Bloch measure of solutions. Our analytical results confirm known physical principles of the transmission problem: The vertical wave number of the incident wave is a conserved quantity. Together with the frequency condition for the transmitted wave, this condition leads (for appropriate photonic crystals) to the effect of negative refraction at the interface.

12:30am - 2:30pm. Lunch.

2:30pm - 3:15pm. *Vincent Pagneux (LAUM, Université du Maine, Le Mans)* - **Control of absorption with resonators by critical coupling.**

When an acoustic wave is reflected by a structured wall with loss, or is both reflected and transmitted through a screen, there is some part of absorption. The perfect absorption case, often desirable, corresponds to the complete annulation of the scattered waves. The method of analysis of acoustic absorption that will be presented uses the structure of the scattering coefficients in the complex frequency plane. In the absence of losses, the scattering coefficient supports pairs of poles and zeros that are complex conjugates and which have imaginary parts linked to the energy leakage by radiation. When losses are introduced and balanced to the leakage, the critical coupling condition is satisfied and perfect absorption is obtained. Several cases will be discussed, using sub-wavelength resonators such as Helmholtz resonators, displaying perfect or quasi-perfect absorption for reflection (one port scattering) or reflection/transmission (two port scattering).

3:15pm - 4:00pm. *Xavier Claeys (LJLL, Université Paris 6), Sonia Fliss (POEMS, ENSTA, Université Paris Saclay), Valentin Vinoles (EPFL, Lausanne, ex-POEMS)* - **Homogenization of transmission problems between homogeneous and periodic half-planes.**

A metamaterial is made of periodic micro-structures with high-contrast coefficients thus the (high-contrast) homogenization theory is well-adapted to describe its effective properties (e.g. negative electromagnetic constants). It is well known that ill-posedness can occur when considering a transmission problem (in the frequency domain) between a dielectric and a negative material due to changes of sign. That is why we want to look carefully at what happens at the interface during the homogenization process for such problems. This question is already relevant for the classical homogenization (without high-contrast) for which “naive” homogenized models lead to non-optimal error estimates. Indeed, such models fail to describe properly the boundary layers induced by the interface. Our objective is to construct approximate effective transmission conditions that restore the optimal estimates (in the case of a transmission problem between a homogeneous and a periodic half-planes, without high-contrast). Following the matched asymptotic method, we propose different asymptotic expansions in different areas, including the classical two-scale ansatz in the periodic half-plane. These new transmission conditions are not standard: they involve Laplace-Beltrami operators at the interface and require to solve cell problems in infinite periodic waveguides. The analysis is based on a combination of Floquet-Bloch Transform and a periodic version of Kondratiev theory. An error estimate is performed and confirms the accuracy of our new model. Numerical results also illustrate its efficiency.

4:00pm - 5:00pm. Coffee Break and Discussions.

6:00pm - 7:00pm. Welcoming Aperitif.

7:30pm - ... Dinner at ”Restaurant le Saint-Jean”, place Saint Jean, Cargese.

9:00 am - 9:45am. *<u>Frédéric Zolla</u> (Institut Fresnel, Marseille)* - **Highly dispersive media.**

The spread of pulse-Doppler radars or short pulse lasers inevitably involves the study of transient regime in dispersive materials. It is indeed well-known that an arbitrary pulse is unaltered when propagating in vacuo. The problem, however, is quite different when dealing with dispersive media. Of course, this problem is a very old stuff and goes back to the early 1900's papers by Sommerfeld and Brillouin and more recently some significant advances have been made by Oughston and in the Fresnel Institute by B. Gralak et al. by using complex and asymptotic analysis techniques. In any case, in highly dispersive materials, the very notion of energy has to be rethought. For instance, the equal distribution of energy between the electric and the magnetic part does not hold anymore. It follows that the notions of velocities (speed of a wave-packet) have to be cautiously handled and especially when dealing with media with real negative permittivity... In addition, the traditional models (Drude's model, Lorentz' model, etc...) for metals or semiconductors are inaccurate for the frequency ranges in nano-photonics. A more general scheme is proposed which preserves some basic properties such as linearity, passivity and causality. Some practical examples will be addressed.

9:45 am - 10:30am. *Houssem Haddar (Defi, Inria, Université Paris Saclay) and <u>Thi Phung Nguyen</u> (Defi, Inria, Université Paris Saclay)* - **Identifying the geometry of a local perturbation in periodic layers.**
We design and analyze sampling methods to recover the support of a local perturbation in a periodic layer from measurements of scattered waves at a fixed frequency. We first present the model problem that corresponds with the semi-discretized problem in the Floquet-Bloch variable of the full locally perturbed periodic problem (we shall explain why this technical assumption is needed in our analysis). We then present the setting of the inverse problem for measurements of scattered waves generated by incident (preparation and evanescent) plane waves. We first show how the Factorization Method and Generalized Linear Sampling Method allow to identify either the background geometry or the background geometry and the defect shape independent from the physical parameters. We then introduce and analyze the sampling methods using a single Floquet-Bloch mode and show how to exploit them to built an indicator function capable of a directly imaging the defect (independently from the background geometry). Numerical validating results on synthetic data will be presented.

10:30 am - 11:00am.- Coffee Break.

11:00 am - 11:45am. *Fioralba Cakoni (Rutgers University), Bojan Guzina (University of Minnesota) and <u>Shari Moskow</u> (Drexel University, Philadelphia)* - **On the homogenization of a scalar scattering problem for highly oscillating anisotropic media.**

We study the homogenization of a transmission problem arising in the scattering theory for bounded inhomogeneities with periodic coefficients modeled by the anisotropic Helmholtz equation. The coefficients are assumed to be periodic functions of the fast variable, specified over the unit cell with characteristic size ε. By way of multiple scales expansion, we focus on the $O(\varepsilon^k)$, $k = 1$, 2, bulk and boundary corrections of the leading-order ($O(1)$) homogenized transmission problem. The analysis in particular provides the H^1 and L^2 estimates of the error committed by the first- order-corrected solution considering (i) bulk correction only and (ii) boundary and bulk correction. We treat explicitly the $O(\varepsilon)$ boundary correction for the transmission problem when the scatterer is a unit square and show it has an L^2-limit as $\varepsilon \to 0$, provided that the boundary cutoff of cells is fixed. We also establish the $O(\varepsilon^2)$ bulk correction describing the mean wave motion inside the scatterer. The analysis also highlights a previously established, yet scarcely recognized, fact that the

$O(\varepsilon)$ bulk correction of the mean motion vanishes identically.

11:45 am - 12:30am. *Mario Ohlberger (Universität Münster) and Barbara Verfürth (Universität Münster)* **- Analysis of a multiscale method for highly heterogeneous Helmholtz problems.**

In this talk, we present and analyze a Heterogeneous Multiscale Method (HMM) for the (two-dimensional) Helmholtz equation with highly heterogeneous coefficient, considering a setting as in [Bouchitté, Felbacq 2004].
We revisit existing homogenization results and combine them to a new two-scale equation. A new stability result for Helmholtz problems with discontinuous coefficients is used to give a wavenumber-explicit bound on the energy norm of the two-scale solution. The HMM is proved to be stable and quasi-optimal under a resolution condition for the macro and fine scale mesh sizes. Finally, we will shortly discuss the reduction of this pollution effect by a two-scale Localized Orthogonal Decomposition, inspired by [Gallistl, Peterseim 2015].
This is work has been supported by the DFG under project number OH 98/6-1.

12:30 am - 2:30pm. Lunch.

2:30pm - 3:15pm. *Pierre Seppecher (IMATH, Université de Toulon)* - **Homogenization of linear elastic structures leading to complete second gradient models.**

The classical homogenization formula describes the limit material as a classical elastic model (a Cauchy material). This formula is often used out of its scope:when the constast between different parts of the material is very high or worse when empty holes are present. On the contrary, we propose cylindrical structures based on a thickened periodic graph and we prove, using the tools of Gamma-convergence and double scale limits, that they have to be described, in the asymptotic limit, as second gradient materials. The choice of the geometry allows us to reduce to the study of discrete systems which correspond to trusses or welded frames. We show how the homogenization of these discrete systems reduces to very simple algebraic formulas.

3:15pm - 4:00pm. *Graeme Milton (University of Utah)* - **Field Patterns: a new mathematical object.**

Here we introduce the theory of field patterns, which is a new mathematical object. We show the results of some initial investigations. This is joint work with Ornella Mattei, and Alexander and Natasha Movchan.

4:00pm - 4:30pm. Coffee Break.

4:30pm - 5:15pm. *Eric Bonnetier (LJK, Université Joseph Fourier, Grenoble), Hoai Minh Nguyen (EPFL, Lausanne)* - **Superlensing using hyperbolic metamaterials.**

We discuss super-lensing in composite media, i.e., the possibility to image an arbitrary object without imposing any conditions on size of the object and the wave length. We are particularly insterested in composites made of inclusions that contain a hyperbolic metamaterial, embedded in a dielectric background: we propose two devices that exhibit superlensing properties.

5:15pm - 6:00pm. Open Question Session and Discussions.

7:00pm - ... Conference Dinner at the Institute.

Friday, November 25th

9:00 am - 9:45am. *Josselin Garnier (CMAP, Ecole Polytechnique, Université Paris Saclay)* - **Effective attenuation in multiscale composite media.**

Experiments show that waves propagating through the earth's crust experience frequency-dependent attenuation. Three regimes have been identified with specific attenuation properties: the low-, mid-, and high-frequency regimes. We show that the observed behavior can be explained via theory for waves in random media. It considers multiple scattering of waves propagating in non-lossy one-dimensional random media with short- and/or long-range correlations. Using stochastic homogenization theory it is possible to show that pulse propagation is described by effective fractional damping exponents, which reproduce well the experimental attenuation properties in the low-, mid-, and high-frequency regimes.

9:45 am - 10:30am. *Jean-Jacques Marigo (LMS, Ecole Polytechnique, Université Paris Saclay), Agnes Maurel (Institut Langevin, ESPCI, Paris), Jean-Francois Mercier (POEMS, CNRS, Université Paris Saclay), Kim Pham (UME, Ensta, Université Paris Saclay)* - **Homogenization of microstructured materials, 2 cases of resonant structures.**
We present homogenization techniques able to encapsulate the resonances of microstructured materials. Two examples of such materials will be considered, which involve different mechanisms of resonance (and thus different approaches of homogenization).

In the former case, we consider a single row of locally resonant inclusions. The resonance is of the Mie type and it is due to a large contrast in the material properties between the inclusions and the matrix. Jump conditions are found involving so-called interface parameters among which one is frequency dependent.

In the later case, we consider resonators of the Helmholtz type, which involve resonance of the Fabry-Perrot type (in the simplest case, quater wavelength resonance). In this case, the homogenization has to account for the propagation within the cavities (homogenization in the bulk) but also it has to account for the boundary layer effects at the neck of the resonator (again, jump conditions are found).

In the two systems, effects of the attenuation will be discussed, and comparisons with direct numerics will be presented.

10:30 am - 11:00am. Coffee Break.

11:00 am - 11:45am. *Hoai Minh Nguyen (EPFL, Lausanne)* - **Negative index materials and their applications.**

In this talk, I will discuss various applications of negative index materials such as cloaking and superlensing. In particular, the possibility that a lens can act like a cloak and conversely is mentioned.

11:45 am - 12:30am. *Maxence Cassier (University of Utah, ex-POEMS), Christophe Hazard (POEMS, CNRS, Université Paris Saclay), Patrick Joly (POEMS, INRIA, Université Paris Saclay)* - **On the limiting amplitude principle for Maxwell's equations at the interface of a metamaterial.**

In this talk, we are interesting in a transmission problem between a dielectric and a metamaterial. The question we consider is the following: does the limiting amplitude principle hold in such a medium? This principle defines the stationary regime as the large time asymptotic behavior of a system subject to a periodic excitation.

An answer is proposed here in the case of a two-layered medium composed of a dielectric and a particular metamaterial (Drude model). In this context, we reformulate the time-dependent Maxwell's equations as a Schrödinger equation and perform its complete spectral analysis. This permits a quasi-explicit representation of the solution via the "generalized diagonalization" of the associated unbounded self-adjoint operator. As an application of this study, we show finally that the limiting amplitude principle holds except for a particular fequency, called the plasmonic frequency, characterized by a ratio of permittivities and permeabilities equal to -1 across the interface. This frequency is a resonance of the system and the response to this excitation blows up linearly in time.

12:30 am - 2:30pm. Lunch.

2:30pm - 3:15pm. *Bérangère Delourme (LAGA, Université Paris 13)* - **Homogenization of a thin perforated wall of finite length.**

This talk deals with the resolution of a scattering problem in a domain made of a thin and periodic layer of finite length placed into an homogeneous medium. The presence of this thin periodic layer of holes is responsible for the appearance of two different kinds of singular behaviors. First, a highly oscillatory boundary layer appears in the vicinity of the periodic layer. Additionaly, since the thin periodic layer has a finite length, corners singularities come up in the neighborhood of its extremities. Based on an approach mixing matched asymptotic expansions and (surface) periodic homogenization, we provide and justify a high order asymptotic expansion which takes into account these two phenomena. Numerical experiments are carried out to illustrate the method.

3:15pm - 4:00pm. *Andrey Pianitsky (Narvik University College, Norway)* - **TBA.**

TBA

4:00pm - ... Coffee Break and Discussions.

7:30pm - ... Dinner at the restaurant "le Serenu", Rue du Colonel Fieschi, Cargese.
