Sujet de Thèse

Non overlapping domain decomposition methods with non local transmission conditions for electromagnetic wave propagation

The numerical simulation of large scale time harmonic wave propagation problems is a challenge in computational science with many applications in engineering sciences . The development of domain decomposition methods (DDM) together with the exploitation of parallel computers led to tremendous improvements in the numerical solution of elliptic partial differential equations, a class of problems to which wave propagation models do not belong. The object of the proposed thesis is to achieve new progress in this field and more particularly on iterative non overlapping domain decomposition methods for solving 3D Helmholtz and time harmonic Maxwell's equations. It is proposed in the framework of the ANR Project NONLOCAL DD that will ensure the funding of the thesis. The thesis will be hosted by the laboratory POEMS (ENSTA, Palaiseau) a joint unit research between CNRS, ENSTA and INRIA. POEMS is one of the partners of the project. The others are the laboratory LJLL of University Paris VI and the team MAGIC3D of INRIA Bordeaux.

The pioneering work of Després [1] then Collino, Ghanemi and Joly [2] and Gander, Magoules and Nataf [3] have shown that it is mandatory, in the context of wave equations, to use impedance type transmission conditions in the coupling of subdomains in order to obtain convergence of the DDM. In the approaches considered so far in the literature, the impedance operator involved in the transmission conditions was always local . However, these methods lead to algebraic convergence of the DDM in the best cases.

In a recent work, F. Collino, P. Joly and M. Lecouvez (in the context of his PhD. thesis of M. Lecouvez, achieved at CEA-Cesta [4]) have observed that using non-local impedance operators such as integral operators with suitable singular kernels (that can be partially localized by an appropriate truncation process) could lead to an exponential convergence of the DDM. One of the strengths of this approach is to rely on a solid theoretical basis that systematically guarantees geometrical convergence, provided that certain properties of injectivity, surjectivity and positivity (in suitable trace spaces) are satisfied by the impedance operator. The developments that we propose hereafter aim at continuing and extending this work (that concerned the scalar Helmholtz equation, mainly in 2D).

The research topic presents various aspects, that concern mathematical analysis, numerical analysis and scientific computing.

- The extension of the above methods to 3D Maxwell's equations. The difficulty is to propose new non local impedance operators adapted to the functional analysis framework for Maxwell's equations (trace spaces of vector fields). In particular, one shall develop the exponentiel convergence theory (before discretization) in the case of heterogeneous media and general (possibly non smooth) non intersecting interfaces. One shall adress the quantitative estimation of the convergence rate in the case of particular geometries (adapted to separation of variables), the influence of both the truncation process and of the frequency on this convergence rate. One shall also address the optimization of the parameters involved in the transmission conditions in this context.

- The study of open questions raised by the thesis of M. Lecouvez, namely
  - The discretization of the transmission conditions and the corresponding numerical analysis. A challenge is to prove the uniform exponential convergence of the iterative algorithm for the discretized problem by showing that the convergence rate is bounded independently of the discretization parameter.
  - The existence of intersecting interfaces (or crossing points in 2D) is known to break down the exponential convergence of the method. A challenge is to design a specific approach to treat these crossing points in order to restore the exponential convergence.
  - Analyze the influence of the number of subdomains on the rate of convergence of the method and propose an approach to minimize its impact (this is related to the question of the scalability of the method).
- The implementation of the method in the parallel code Montjoie developed by M. Duruflé (INRIA Bordeaux), which includes the optimization of various algorithmic aspects
  - The efficient numerical evaluation by appropriate quadrature of the entries of the coupling matrices between subdomains,
  - The algorithms for the acceleration of the inversion of these coupling matrices (for instance H matrix methods),
  - The implementation, the comparison and possibly the analysis of various iterative algorithms (Jacobi, Gauss-Seidel, GMRES, ...) method.

The ideal profile of the candidate would be someone with solid background in the theory of partial differential equations and their numerical analysis as well as competences in scientific computation and programming. However, the subject is broad enough to be oriented according to the preferences and abilities of the candidate.

References

[1] Després B. Domain decomposition method and the Helmholtz problem (Part II) In Second international conference on mathematical and numerical aspect of wave propagation phenomena, SIAM 1993, p. 197-206

[2] Collino F., Ghanemi S., Joly P. Domain decomposition method for harmonic wave propagation: a general presentation, CMAME, Vol. 184, 24, 2000, p. 171-211

[3] Gander, M., Magoulès, F., and Nataf, F. Optimized Schwarz Methods without Overlap for the Helmholtz Equation SIAM Journal on Scientific Computing, 2002, Volume 24:1, Pages 38-60

[4] Lecouvez, M. Méthodes itératives de décompostion de domaine sans recouvrement avec convergence géométrique pour l'équation de Helmholtz, Thse de l'École Polytechnique, Juillet 2015

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