Open position for an Internship at *POEMS*

UMR 7231 CNRS/INRIA/ENSTA

2013-2014 Academic year

The Method of Reduction of the Dimension: Studies and numerical experiments

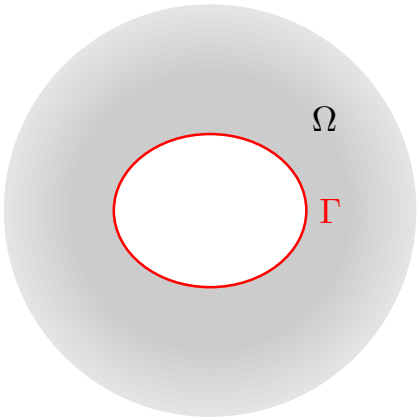
Keywords: boundary element method, singular integrals, homogeneous functions.

▷ **Scientific context.**

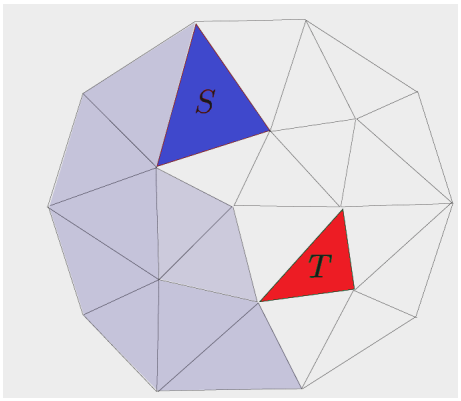
The Boundary Integral Equations are well adapted to solve problem in infinite domain since it transform a problem defined in a domain Ω in an integral equation on the boundary of the domain $\Gamma = \partial\Omega$ (see Figure 1(a)). The numerical resolution of such integral equations can be done using the Boundary Element Method (BEM, see [1]). For the resolution, we introduce a mesh of Γ (see Figure 1(b)). A difficulty of the BEM is that it requires the evaluation of integrals with singular integrands on the product of two elements of the mesh (for example triangles). For the Laplace equation, one may have to evaluate the following integrals:

$$I = \int_{S\times T} \frac{\psi(y)\phi(y)}{\|x-y\|} dxdy, \text{ or } J = \int_{S\times T} \psi(y) \frac{(x-y)}{\|x-y\|^3} \phi(x) dxdy \qquad (1)$$

where S and T are triangles from the mesh and ψ and ϕ are basis functions.



(a) Domain Ω and its boundary Γ.



(b) Mesh of Γ.

The POEMS Team (Propagation des Ondes: Etudes Mathematiques et Simulation) at ENSTA-ParisTech has developed a new method to evaluate singular integrals arising in the BEM (see [2]), this method is called the Method of Reduction of the Dimension (MRD) since it relies upon the reduction of the dimension of the integration domain to reduce 4-D integrals into a linear combination of 1-D integrals (see [3] and [4]).

▷ **Objectives**

The MRD requires to distinguish several geometrical configuration of the triangles so the first point is to study the limit of the calculation for triangles in secant planes which become parallel. Several other interesting and challenging points which can be adapted with the trainee are : study of the best approach in term of simplicity of implantation between the analytic MRD and reductions combined

with numerical integration or working on a small code to solve Laplace equation and compare the impact on the accuracy of the result using the MRD or other methods (for example the method presented in [1]).

- ▷ **Knowledge.**
  A background in Partial Differential Equations and in the Finite Element Method are expected in order to understand the basis of Boundary Element Methods.

- ▷ **Contact and location**
  The internship will take place in the POEMS team at **ENSTA ParisTech** at Palaiseau in France, on the campus of the Ecole Polytechnique under the supervision of **Marc Lenoir** and in collaboration with Nicolas Salles. The expected duration is **3-5 months**. Applications including CV and cover letter have to be sent by email.

  The contacts are:

  - ○ **Marc Lenoir**
    Unité de Mathématiques Appliquées, ENSTA ParisTech, Palaiseau
    email: marc.lenoir@ensta-paristech.fr

  - ○ **Nicolas Salles**
    Department of Mathematics, University College London, London
    email: n.salles@ucl.ac.uk

The position is open to international students

References

[1] Sauter, S. A., and Schwab, C. *Boundary element methods.* Springer, 2011

[2] Lenoir, M. *Influence coefficients for variational integral equations.* Comptes Rendus Mathematique, 2006.

[3] Lenoir, M., and Salles, N. *Evaluation of 3-D Singular and Nearly Singular Integrals in Galerkin BEM for Thin Layers.* SIAM Journal on Scientific Computing, 2012.

[4] Salles, N. *Calcul des singularités dans les méthodes d'équations intégrales variationnelles.* Thèse de doctorat de l'université Paris Sud, 2013.