Wind farm cable layout optimization with constraints of load flow and robustness

Cédric Bentz, Marie-Christine Costa, Pierre-Louis Poirion, Thomas Ridremont, and Camille Zakour,

Abstract—We consider an offshore wind farm defined by the location of the wind turbines and the amount of energy supplied per turbine, as well as the location of the central station responsible for redistributing the collected energy to users on the power grid. Knowing the power injected at each node, the capacities, susceptance and costs of the cables that can be used, the goal is to determine the least expensive cabling to route the energy supplied by the wind turbines to the central station. This cabling must respect the capacity constraints on the cables as well as the electrical constraints of *Load Flow* defined at each node of the network. In a second step, we look for a cabling that is also robust in case of failure of a cable, the notion of robustness being seen here as the protection again the worst case of failure. This work was carried out in collaboration with EDF Energies renouvelables.

We give a mathematical model of the problem taking into account all the constraints of capacity, connectivity, load flow, cable types and incompatibility between edges, in the form of a mixed integer quadratic program that can be linearized and solved using a MIP solver. We then propose two mathematical models for the robust problem, formulation inspired by the previous one and a bi-level program where the second level is a max min program. Finally we present the results of our tests which provide solutions for real data up to about 50 nodes, before concluding.

Keywords—IEEEtran, Operational Research, Bilevel Optimization, Robust Optimization, Renewable Energy.

I. Introduction

A. Presentation of the problem

Optimizing a renewable energy system is an important topic nowadays [9] and the design of wind farms involves many challenges in optimization ([7], [10]). More specifically, the search for an optimal wiring of wind farms has been recently investigated ([3],[6], [8], [12], [15]), but to our knowledge without taking into account simultaneously load flow and robustness constraints. More generally, the design of resilient network is a today's research topic ([2], [5]).

In this paper, we assume that one of the cables can fail and we consider the problem of designing a robust cabling network of an offshore wind-farm, at minimal cost, once the location of the turbines has already been decided. More precisely, given a set of offshore wind turbines producing a known quantity of energy, we look for the cheapest network able to route the energy produced by all the wind turbines to the point of common coupling (PCC), called *root node* thereafter, that will collect the energy and dispatch it to the grid. One of the main characteristics of our network is that it should be *robust*, i.e. resilient to the failure of a cable: hence, it should be able to route, for any possible breakdown, all the produced energy from the wind turbines to the root node. An important constraint of our problem is that the flow of energy routed in the network must satisfy the *Load Flow equations*.

We model the problem by using an undirected graph $G = (V, E)$ representing the given support network. In offshore wind-farm, the graph G is generally the union of a partial grid on n nodes, of some diagonal edges and of the root node $r \in V$ linked to a subset of nodes of the grid. The set of wind turbines is denoted by $T \subset V \setminus \{r\}$. Hence $V \setminus (T \cup \{r\})$ denotes the set of junction nodes. The location of the root, of the wind turbines and of the possible junction nodes are known, so the lengths of the edges of E are given. There are Q different types of electrical cables numbered by $q \in [1, ..., Q]$, and Q is generally a small number (≤ 3). For each $q \in [1, ..., Q]$ and each $[v, w] \in E$, we denote by c^q_{vw}, the cost of installing a cable of type q on $[v, w]$. This cost depends on the type of cable chosen and on the length of $[v, w]$. The capacity of a cable of type q on $[v, w]$ is denoted by u^q_{vw} ($c^q_{vw} = c^q_{wv}$ and $u^q_{vw} = u^q_{wv}$). Hence, we aim to design the cheapest sub-network of G spanning $T \cup \{r\}$, such that the capacity on each installed cable is greater than the active power flow routed from the terminals to the root node through this cable, i.e. $\Pi_{uv} \leq u^q_{uv}$ and $\Pi_{vu} \leq u^q_{uv}$ for each edge $[u, v]$ where a cable of type q is installed. See Figure 1 for an example where the production of each turbine is equal to 1 and the capacity of each cable equal to 2.

For technical constraints given by EDF, there is a set $\mathcal{I} \subset E \times E$ of pairs of edges $\{e, e'\}$ such that it is not allowed to install a cable on both e and e'. In practice, the set $\mathcal{I}$ is used to avoid installing cables on two edges that intersect each other, and hence to ensure that the resulting network is planar (on Figure 1, we would have for example $\{e_1, e_2\} \in \mathcal{I}$).

In this paper, we first explain briefly the *Load Flow* equations and how to take them into account. Then we formulate the problem when no breakdown can occur as a mixed-integer linear program which can be solved exactly by using a MIP solver. Then we propose two mathematical formulations for the robust case: the first one, derived from the problem without any breakdown, is a mixed-integer linear program; the second one is a bi-level mixed integer linear program which is a compact formulation. The number of constraints and variables can be huge in the robust case and we propose to use a constraint generation algorithm ([1]) to

Cédric Bentz and Marie-Christine Costa are at CEDRIC, CNAM, 292 rue Saint-Martin 75003, Paris, France. Marie-Christine Costa also belongs to ENSTA IP-Paris.

Pierre-Louis Poirion is at RIKEN Center for Advanced Intelligence Project, Tokyo, Japan.

Thomas Ridremont is at ARTELYS, Paris, France.

Camille Zakour is at EDF Power Systems Engineering Center, Saint-Denis, France.

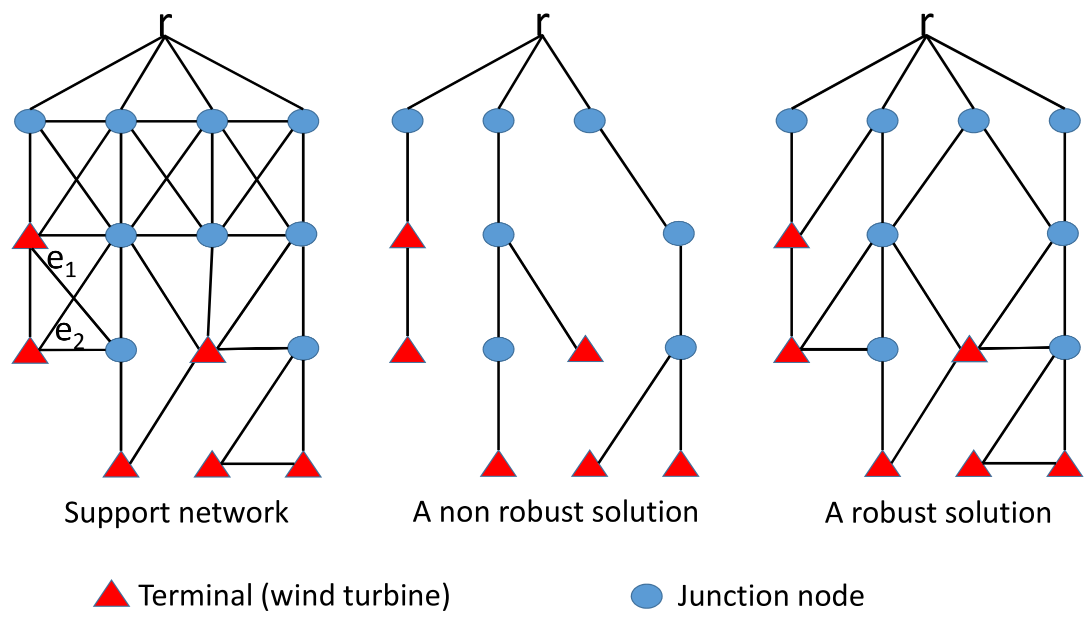

Fig. 1. Support network and solutions on a grid 4×4

solve the problem. Finally, we give some results obtained on real data before concluding.

B. The Load Flow problem

1) Approximation of the load flow equations: Load flow studies, also known as power flow studies, are power system analysis. We briefly explain them in this section but we refer the reader to [14] for more information about electrical power system planning. We define a bus as a node of an electrical network (for example circuit breakers, transformers, conductors or capacitors): in our application, a node is either a junction node or a wind turbine or the substation.

Given the production capacities of the different generators (here, the wind turbines), the load flow analysis is a non-linear system which determines voltage magnitude and phase angle at each node, and the real and reactive power flowing through each line of the network. In order to introduce the load-flow constraints in the optimization model, we consider the *Direct Current* (DC) estimations of the load flow for which it is customary to make the following assumptions [14]:

- Line resistances (including active power losses) are negligible
- Phase angle differences are small
- Magnitudes of node voltages are equal to 1.0 per unit

Let P_v denote the given net active power injection at node v, θ_v denotes the phase angle in radians at node v and B_{vw} denotes the given susceptance, physical quantity related to the cable, on the line $[v,w]$, with $B_{vw} = 0$ if no line is built on $[v,w]$. Then the DC approximation of the load flow is given by:

$$P_v = \sum_{w=1}^{n} B_{vw}(\theta_v - \theta_w) \tag{1}$$

2) Integration of the Load Flow equations in our problem: Regarding the design of a wind farm and the associated graph $G = (V,E)$, we are given for each edge $[v,w] \in E$ a susceptance B_{vw}. However, if we do not build a cable on the edge $[v,w]$ in the final network, we can set B_{vw} to 0. We also consider $\vec{G} = (V,\vec{E})$ which is the bi-directed graph associated with G, i.e. for each edge $[v,w] \in E$ in G, there are the arcs $\{(v,w),(w,v)\} \subset \vec{E}$ in $\vec{G}$.

For each node v different from the root node (i.e. $v \in V \setminus \{r\}$), it is known that $P_v \geq 0$: either $P_v > 0$ if v provides some power injection in the network (i.e. v is a wind turbine in our case), or $P_v = 0$ if v is a junction node. If v is a wind turbine, P_v is known and gives the estimation of the production of energy of the wind turbine v. For the root r, the load flow equations imply that $P_r = -\sum_{v \in V\setminus\{r\}} P_v$. For each $(v,w) \in \vec{E}$, we define:

$$\Pi_{vw} := B_{vw}(\theta_v - \theta_w)$$

as the active power flow through $[v,w]$ from v to w, and thus we have $\Pi_{vw} = -\Pi_{wv}$ and $P_v = \sum_{w=1}^{n} \Pi_{vw}$.

Proposition 1. *If the load flow equations (1) are satisfied at each node of a subgraph $\hat{G} = (V, \hat{E})$ of $G = (V, E)$, with $\hat{E} \subseteq E$ and B_{vw} set to 0 if no cable is built on the edge $[v, w]$ (i.e. $[v, w] \notin \hat{E}$), then there exists a chain in $\hat{G}$ between each node v producing a positive active power flow (i.e. such that $P_v > 0$) and the root.*

Proof: Assume that there exists $v' \in V$ such that $P_{v'} > 0$ and v' is not connected to the root node r. Let $G(v') = (V', E')$ be an inclusion-wise maximal connected subgraph of G that contains v'. By assumptions $r \notin V'$. Let us consider the sum

$$S = \sum_{v \in V'} P_v$$

Since $P_{v'} > 0$, $P_v \geq 0 \quad \forall v \in V \setminus \{r\}$, and $r \notin V'$, we have $S > 0$. However, using Equation (1), we also have:

$$S = \sum_{v \in V'} P_v = \sum_{v \in V'} \sum_{w \in V'} \Pi_{vw} = 0$$

since $\Pi_{vw} = -\Pi_{wv}$. Hence a contradiction. ■

Proposition 1 ensures that if Constraints (1) are satisfied in a mathematical program, we do not have to add connectivity constraints to ensure that there exists a path between the root and each wind turbine, since load flow equations will not be satisfied otherwise.

II. A FORMULATION FOR THE PROBLEM WITHOUT BREAKDOWNS

In this section, we present the problem when no breakdown can occur on the cables and we give a mixed-integer linear model to solve the problem. In the following, we are given P_v for every node different from the root node, and $P_r = -\sum_{v \in V \setminus \{r\}} P_v$.

Recall that the power injection P_v at node $v \in V$ is given for each node v. Moreover, B^q_{vw}, corresponding to the susceptance of a cable of type q installed between v and w, is also given for each $[v, w] \in E$ and each $q \in [1, .., Q]$.

We introduce the following variables:

- For each $q \in [1, ..., Q]$ and for each $e = [v, w] \in E$, let y^q_e be the binary variable such that $y^q_e = 1$ if and only if a cable of type q is installed on $e = [v, w]$. Notice that, in the following, for each $e = [v, w] \in E$, y^q_e can be written indifferently y^q_e, y^q_{vw} or y^q_{wv}.
- For each $v \in V$, let θ_v be the voltage angle at v.

We aim to minimize the total cost of the resulting network, i.e.:

$$\min \sum_{e \in E} \sum_{q=1}^{Q} c^q_e y^q_e$$

We now give the different constraints associated with our problem:

a) Cable types constraints:
: For each $e \in E$, we cannot install more than one type of cable:

$$\sum_{q=1}^{Q} y^q_e \leq 1, \quad \forall e \in E \tag{2}$$

b) Constraints of incompatibility between edges:
: The set $\mathcal{I} \subset E \times E$ contains every pair $\{e_1, e_2\}$ such that there cannot be a cable on both edges e_1 and e_2:

$$\sum_{q=1}^{Q} (y^q_{e_1} + y^q_{e_2}) \leq 1, \ \forall (e_1, e_2) \in \mathcal{I} \tag{3}$$

We also introduce $E_{\mathcal{I}}$ the set of edges e with at least one constraint of incompatibility with another edge, i.e. $e \in E_{\mathcal{I}}$ if there exists at least one edge e' such that $\{e, e'\} \in \mathcal{I}$.

c) Voltage angles constraints:
: In the load flow equations, we only consider the differences of angles $\theta_v - \theta_w$ for each pair $(v, w) \in V^2$ with $v \neq w$, i.e. the value of θ_v alone is useless. Using this fact and Assumption B, which states that voltage angles are assumed to be small, we can fix arbitrarily and without loss of generality the value of the angle at the root node r,

$$\theta_r = 0, \tag{4}$$

and for all $v \in V$, $-\varepsilon \leq \theta_v - \theta_r \leq \varepsilon$ for some small $\varepsilon > 0$, which implies:

$$-\varepsilon \ \leq \ \theta_v \ \leq \ \varepsilon \qquad \forall v \in V$$

Differences of voltage angles are assumed to be less than 10^{-1}, hence we have $\varepsilon \approx 10^{-1}$.

d) Load Flow constraints:
: For all $v \in V$, the load flow equations are given by:

$$P_v \quad = \quad \sum_{w:[v,w] \in E} \sum_{q=1}^{Q} B^q_{vw} y^q_{vw} (\theta_v - \theta_w) \tag{5}$$

where

$$\sum_{q=1}^{Q} B^q_{vw} y^q_{vw} (\theta_v - \theta_w) \quad = \quad \Pi_{vw}$$

corresponds to the active power sent through $(v, w) \in \vec{E}$. Using Property 1, Constraints (5) ensure the connectivity between the root and the wind turbines.

e) Capacity constraints:
: For each $e \in E$, the power flow Π_e routed through e must be smaller than the capacity of the cable installed on e:

$$\Pi_{vw} = \sum_{q=1}^{Q} B^q_{vw} y^q_{vw} (\theta_v - \theta_w) \quad \leq \quad \sum_{q=1}^{Q} u^q_{vw} y^q_{vw} \qquad \forall (v, w) \in \vec{E} \tag{6}$$

One can notice that, if $\sum_{q=1}^{Q} B^q_{vw} y^q_{vw} (\theta_v - \theta_w) < 0$ (resp. > 0), then $\sum_{q=1}^{Q} B^q_{wv} y^q_{wv} (\theta_w - \theta_v) > 0$ (resp. < 0), depending on whether the energy is routed from v to w or from w to v.

The mathematical program to solve can be written as follows:

$(\mathbf{LFF})$:

$$\min_{y,\theta} \quad \sum_{e\in E}\sum_{q=1}^{Q} c_e^q y_e^q$$

$$\text{s.c.} \quad \sum_{q=1}^{Q} y_e^q \le 1,\ \forall e \in E \setminus E_{\mathcal{I}} \tag{2}$$

$$\sum_{q=1}^{Q}(y_{e_1}^q + y_{e_2}^q) \le 1,\ \forall (e_1,e_2) \in \mathcal{I} \tag{3}$$

$$\theta_r = 0, \tag{4}$$

$$\sum_{w:[v,w]\in E}\sum_{q=1}^{Q} B_{vw}^q y_{vw}^q(\theta_v - \theta_w) = P_v,\ \forall v \in V \setminus \{r\} \tag{5}$$

$$\sum_{q=1}^{Q} B_{vw}^q y_{vw}^q(\theta_v - \theta_w) \le \sum_{q=1}^{Q} u_{vw}^q y_{vw}^q,\ \forall (v,w) \in \vec{E} \tag{6}$$

$$y \in \{0,1\}^{|E|Q}, \quad \theta \in [-\varepsilon,\varepsilon]^{|V|}$$

The above program have non-linear terms, $y_{vw}^q\theta_v$ and $y_{vw}^q\theta_w$ in Constraints (5) and (6). Since $-\varepsilon \le \theta_v \le \varepsilon\ \forall v \in V$, we have that $0 \le \theta_v + \varepsilon \le 2\varepsilon$ for each $v \in V \setminus \{r\}$. To deal with such term, we use a well-known technique consisting in linearizing each term $y_{vw}^q(\theta_v+\varepsilon)$ which is the product of a binary variable by a non-negative bounded real variable, see for instance [4]. For all $(v,w) \in \vec{E}$, $q \in [1,...,Q]$, we introduce a non-negative variable ρ_{vw}^q satisfying the constraints below:

$$\rho_{vw}^q \quad \le \quad \theta_v + \varepsilon\ \forall (v,w) \in \vec{E},\ \forall q \in [1,..,Q] \tag{7}$$

$$\rho_{vw}^q \quad \le \quad 2\varepsilon y_{vw}^q\ \forall (v,w) \in \vec{E},\ \forall q \in [1,..,Q] \tag{8}$$

$$\rho_{vw}^q \quad \ge \quad \theta_v + \varepsilon - 2\varepsilon(1 - y_{vw}^q)\ \forall (v,w) \in \vec{E},\ \forall q \in [1,..,Q] \tag{9}$$

We know introduce the following polyhedron $\mathcal{L}(\theta,y)$,

$$\mathcal{L}(\theta,y) = \left\{\rho \in \mathbb{R}^{|\vec{E}|Q} \mid \ (7),\ (8),\ (9)\right\}$$

We have:

$$\begin{aligned} &\rho \ge 0,\ \rho \in \mathcal{L}(\theta,y) \\ &\Rightarrow\ \rho_{vw}^q = y_{vw}^q(\theta_v + \varepsilon)\forall (v,w) \in \vec{E},\ \forall q \in [1,..,Q] \end{aligned}$$

Since we have $y_{vw}^q(\theta_v - \theta_w) = y_{vw}^q\theta_v - y_{vw}^q\theta_w = y_{vw}^q(\theta_v + \varepsilon) - y_{vw}^q(\theta_w + \varepsilon)$, we also have

$$y_{vw}^q(\theta_v - \theta_w) = \rho_{vw}^q - \rho_{wv}^q$$

.

We can then linearize the mathematical program $(\mathbf{LFF})$ and we obtain:

$(\mathbf{LLFF})$:

$$\min_{y,\theta,\rho} \quad \sum_{e\in E}\sum_{q=1}^{Q} c_e^q y_e^q$$

$$\text{s.t.} \quad \sum_{q=1}^{Q} y_e^q \le 1,\ \forall e \in E \setminus E_{\mathcal{I}} \tag{2}$$

$$\sum_{q=1}^{Q}(y_{e_1}^q + y_{e_2}^q) \le 1,\ \forall (e_1,e_2) \in \mathcal{I} \tag{3}$$

$$\theta_r = 0 \tag{4}$$

$$\sum_{q=1}^{Q}\sum_{w:[v,w]\in E} B_{vw}^q(\rho_{vw}^q - \rho_{wv}^q) = P_v, \forall v \in V \setminus \{r\} \tag{5}$$

$$\sum_{q=1}^{Q} B_{vw}^q(\rho_{vw}^q - \rho_{wv}^q) \le \sum_{q=1}^{Q} u_{vw}^q y_{vw}^q, \forall (v,w) \in \vec{E} \tag{6}$$

$$\begin{aligned} &\rho \in \mathcal{L}(\theta,y), \rho \ge 0 \\ &y \in \{0,1\}^{|E|Q}, \quad \theta \in [-\varepsilon,\varepsilon]^{|V|} \end{aligned}$$

A solution to $(\mathbf{LLFF})$ can easily be obtained by using a MILP software.

III. Formulations for the robust problem

We consider now that a breakdown may occur on one of the installed cables. We assume that the support network always admit a robust solution, for instance by installing a cable on all the edges.

A. A mixed-integer linear formulation

We use the same binary variables y_e^q, for each $q \in \{1, ..., Q\}$ and $e \in E$ as in Section II and we denote by $\xi \in E$ the cable where the breakdown occurs. We also introduce the following variables: for each $v \in V$ and each $\xi \in E$, θ_v^ξ is the voltage angle at v in the network when there is a breakdown on the cable built on the edge ξ, and $\vec{\xi}$ is the set of bi-directed arcs associated with ξ. We propose the following mathematical program $(\mathbf{ROB})$ to design an optimal robust network:

$$(\mathbf{ROB}):$$

$$\min_{y,\theta} \sum_{e\in E}\sum_{q=1}^{Q} c_e^q y_e^q$$

$$\text{s.t.} \sum_{q=1}^{Q} y_e^q \leq 1, \forall e \in E \setminus E_{\mathcal{I}} \tag{10}$$

$$\sum_{q=1}^{Q}(y_{e_1}^q + y_{e_2}^q) \leq 1,\ \forall (e_1, e_2) \in \mathcal{I} \tag{11}$$

$$\theta_r^\xi = 0 \forall \xi \in E \tag{12}$$

$$\sum_{q=1}^{Q}\sum_{[v,w]\in E\setminus\{\xi\}} B_{vw}^q y_{vw}^q(\theta_v^\xi - \theta_w^\xi) = P_v, \quad \forall v \in V \setminus \{r\},\ \forall \xi \in E \tag{13}$$

$$\sum_{q=1}^{Q} B_{vw}^q y_{vw}^q(\theta_v^\xi - \theta_w^\xi) \leq \sum_{q=1}^{Q} u_{vw}^q y_{vw}^q \quad \forall \xi \in E,\ \forall (v,w) \in \vec{E} \setminus \{\vec{\xi}\} \tag{14}$$

$$y \in \{0,1\}^{|E|Q}, \theta \in [-\varepsilon, \varepsilon]^{|V||E|}$$

Constraints (10) and (11) are identical to (2) and (3), while Constraints (12) ensure that θ_r is equal to 0 for each case of breakdown $\xi \in E$. Constraints (13) ensure that the load flow is respected (and so each turbine is connected to the root) for any breakdown $\xi \in E$ by not considering the active power on ξ in this case. Constraints (14) ensure that, for any breakdown $\xi \in E$, the capacities in the resulting network are high enough to support the active power through the cables.

Again, we have a product of variables θ and y, which we linearize in a similar way as in the non-robust model: $\mathcal{L}^\xi(\theta, y)$, is the set of linearization constraints where $\rho_{vw}^{q,\xi} = y_{uv}^q(\theta_v^\xi + \varepsilon)$ for all $(v, w) \in \vec{E}$, $q \in Q$ and $\xi \in E$. The linearized problem becomes:

$$(\mathbf{LPROB})$$

$$\min_{y,\theta,\rho} \sum_{e\in E}\sum_{q\in\mathcal{Q}} c_e^q y_e^q$$

$$\text{s.c.} \sum_{q=1}^{Q} y_e^q \leq 1, \forall e \in E \setminus E_{\mathcal{I}} \tag{10}$$

$$\sum_{q=1}^{Q}(y_{e_1}^q + y_{e_2}^q) \leq 1,\ \forall (e_1, e_2) \in \mathcal{I} \tag{11}$$

$$\theta_r^\xi = 0 \forall \xi \in E \tag{12}$$

$$\sum_{q=1}^{Q}\sum_{[v,w]\in E\setminus\{\xi\}} B_{vw}^q(\rho_{vw}^{q,\xi} - \rho_{wv}^{q,\xi}) = P_v, \quad \forall v \in V \setminus r,\ \xi \in E \tag{13}$$

$$\sum_{q=1}^{Q} B_{vw}^q(\rho_{vw}^{q,\xi} - \rho_{wv}^{q,\xi}) \leq \sum_{q=1}^{Q} u_{vw}^q y_{vw}^q \forall (v,w) \in \vec{E}, \xi \in E \tag{14}$$

$$\rho \in \mathcal{L}^\xi(\theta, y)$$

$$y \in \{0,1\}^{|E|Q}, \theta \in [-\varepsilon, \varepsilon]^{|V||E|}, \rho \geq 0$$

The number of variables and constraints of $(\mathbf{LPROB})$ can be very high depending on the size of the graph. We propose a cutting plane algorithm to deal with this case. We initialize $(\mathbf{LPROB})_{E_S}$ which corresponds to $(\mathbf{LROB})$ with only a small subset of edges $E_S \subset E$ in Constraints (12)-(14), i.e. we define those constraints only for $\xi \in E_S$. We begin to solve the reduced problem $(\mathbf{LPROB})_{E_S}$. We define the set of edges selected in the current integer solution $(\hat{y}, \hat{\theta}, \hat{\rho})$: $\hat{E}_C = \{e \in E \mid \sum_{q=1}^{Q} \hat{y}_e^q = 1\}$ and $\vec{E}_C$ corresponds to the set of bi-directed arcs associated to $\hat{E}_C$. We define $\hat{E}_S = E_S \cap \hat{E}_C$ corresponding to the intersection between the set of edges which are in the solution $(\hat{y}, \hat{\theta}, \hat{\rho})$ and the set of edges for which the scenario of breakdown has been taken into account at this moment in the algorithm. Finally, we define:

$$\hat{B}_{vw} = \sum_{q=1}^{Q} B_{vw}^q \hat{y}_{vw}^q \quad \text{and} \quad \hat{u}_{vw} = \sum_{q=1}^{Q} u_{vw}^q \hat{y}_{vw}^q \quad \forall (v,w) \in \vec{E}_C$$

where $\hat{B}_{vw}$ (respectively $\hat{u}_{vw}$) corresponds to the susceptance (respectively capacity) on the cable built on $[v, w]$. For the integer feasible solution $(\hat{y}, \hat{\theta}, \hat{\rho})$, we introduce the following sub-problem $(\mathbf{SUB})_e$ for each $e \in \hat{E}_C \setminus \hat{E}_S$:

$(\mathbf{SUB})_e$

$$\min_{\theta} 0$$

$$\text{s.c.} \quad \theta_r = 0 \tag{15}$$

$$\sum_{w:[v,w]\in E_C\setminus\{e\}} \hat{B}_{vw}(\theta_v - \theta_w) = P_v, \ \forall v \in V \setminus r \tag{16}$$

$$\hat{B}_{vw}(\theta_v - \theta_w) \ \leq \ \hat{u}_{vw}, \ \forall (v,w) \in \vec{E}_C \setminus \{e\} \tag{17}$$

$$\theta \in [-\varepsilon, \varepsilon]^{|V|}$$

The formulation $(\mathbf{SUB})_e$ allows to determine whether if the load-flow constraints are still satisfied even if we remove the edge e from the current integer solution, i.e. to ensure that the solution is resilient to a breakdown on e. We already ensure that the energy is still routed to the sub-station even in the event of a breakdown on any edge in $\hat{E}_S$, and we have to ensure that this is the case for edges in $\hat{E}_C \setminus \hat{E}_S$. When an integer solution better than the current one is found, we solve the set of sub-problems $(\mathbf{SUB})_e$ for $e \in \hat{E}_C \setminus \hat{E}_S$. If one subproblem $(\mathbf{SUB})_{\bar{e}}$ for a given $\bar{e} \in \hat{E}_C \setminus \hat{E}_S$ does not have any feasible solution, we add $\bar{e}$ to E_S and we add the constraints associated to $(\mathbf{LROB})_{E_S}$. Otherwise, if all subproblems have feasible solutions, the integer solution is feasible for the general problem. Please note that the subproblems $(\mathbf{SUB})_e$ have only continuous variables and so are easy to solve.

B. Bilevel formulation

Bilevel formulations are a good approach for modeling network design optimization [11]. The bilevel formulation proposed here is particular in that the second level is a $\max\min$ problem: it can be seen as a game with a defender who makes his decisions in two steps and an attacker who intervenes between these two decisions.

The variables y and θ are defined as in Section III-A. For each $[i,j] \in E$, we introduce a binary variable b_{ij} where $b_{ij} = 1$ if and only if the attacker chooses to delete the edge $[i,j]$. We also introduce the variables η_v for each $v \in V\setminus\{r\}$, which correspond to penalty variables used to satisfy the load flow equations. We define the following polyhedron:

$$\mathcal{B}(y) = \{ \ b \in \{0,1\}^{|E|} \mid \textstyle\sum_{[i,j]\in E} b_{ij} = 1 \ ; \ b_{ij} \leq \sum_{q\in Q} y^q_{ij} \quad \forall [i,j] \in E \ \}$$

which defines the set of possible scenarios of edge deletions (i.e. the set of constraints of the attacker): the deleted edge must belong to the ones selected by the defender. We also introduce for each $[i,j] \in E$ and $q \in Q$ the notation

$$\beta^q_{ij} = B^q_{ij}(y^q_{ij} - b_{ij})$$

where β_{ij} is equal to B^q_{ij} if a cable of type q is built by the defender on the edge $[i,j]$ and not deleted by the attacker, and to 0 otherwise (since we have $b_{ij} \leq \sum_{q\in Q} y^q_{ij}$).

We then define the following polyhedron:

$$\mathcal{X}(y,b) = \{$$

$$\theta_r \ = \ 0 \tag{18}$$

$$\sum_{q=1}^{Q} \sum_{[v,w]\in E} \beta^q_{vw}(\theta_v - \theta_w) + \eta_v = P_v, \forall v \in V \setminus r \tag{19}$$

$$\beta^q_{vw}(\theta_v - \theta_w) \ \leq \ u^q_{vw} \quad \forall (v,w) \in \vec{E}, \ \forall q \in Q \tag{20}$$

$$\eta \in \mathbb{R}^{|V|-1}_+, \theta \in [-\varepsilon, \varepsilon]^{|V|}\}$$

The polyhedron $\mathcal{X}(y,b)$ corresponds to the set constraints (12), (13) and (14) for given values of y and b, i.e. once the network has been built and the attacker has deleted an edge. Then, β corresponds to the susceptances in the residual network defined by (y,b). The variables η_v are penalty variables which ensure that the polyhedron is non-empty: the solution where we have $\theta_v = 0$ and $\eta_v = P_v$ for each node v is always feasible. The load flow and capacity constraints are satisfied if there exists a feasible solution with $\sum_{v\in V} \eta_v = 0$.

We propose the following bilevel program:

$(\mathbf{BIL})$:

$$\min_{y\in\{0,1\}^{|A|}} \sum_{(i,j)\in A} c_{ij} y_{ij}$$

$$\text{s.t.} \quad \sum_{q=1}^{Q} y^q_e \leq 1, \forall e \in E \setminus E_{\mathcal{I}} \tag{21}$$

$$\sum_{q=1}^{Q} (y^q_{e_1} + y^q_{e_2}) \leq 1, \ \forall (e_1, e_2) \in \mathcal{I} \tag{22}$$

$$f(y) = 0 \tag{23}$$

$$\text{where } f(y) = \max_{b\in\mathcal{B}(y)} \min_{(\theta,\eta)\in\mathcal{X}(y,b)} \sum_{v\in V} \eta_v \tag{24}$$

$$y \in \{0,1\}^{|E|Q} \tag{25}$$

The defender first builds a network considering the constraints on the types of cables and the planarity constraints given by Constraints (21) and (22) respectively. Let $\hat{y}$ be the solution associated to this network. Then, the attacker deletes an edge that the defender has built. Finally, the defender verifies that the load flow and capacity constraints are still satisfied: he minimizes $\sum_{v\in V} \eta_v$, which is a sum of positive variables and $\hat{y}$ is a feasible solution if and only if $\sum_{v\in V} \eta_v = 0$. Recall that there is at least one solution which is to set $y_{ij} = 1$ for all $[i,j] \in E$. The defender search for the minimum cost feasible solution.

Let $(\mathbf{PL}_\eta)$ denote the min problem of the second level: $\min_{(\theta,\eta)\in\mathcal{X}(y,b)} \sum_{v\in V} \eta_v$.

At this step, y and b are fixed and so β is fixed too, thus $(\mathbf{PL}_\eta)$ is a linear program bounded by 0 since $\eta_v \geq 0$ for all v and it can be dualized. Let κ, λ and μ denote the dual variables associated respectively to constraints (18), (19) and (20) and let $\mathcal{DX}(y,b)$ be the the dual polyedron, not detailed here, defined by the dual constraints of $\mathcal{X}$. Then $(\mathbf{PL}_\eta)$ can be written as

$$\max_{\kappa,\lambda,\mu\in\mathcal{DX}(y,b)} \sum_{v\in V\setminus\{r\}} P_v\lambda_v + \sum_{(v,w)\in\vec{E}}\sum_{q\in Q} u^q_{vw}\mu^q_{vw}$$

and we have:

$$f(y) = \max_{b\in\mathcal{B}(y),\kappa,\lambda,\mu\in\mathcal{DX}(y,b)} \sum_{v\in V\setminus\{r\}} P_v\lambda_v + \sum_{(v,w)\in\vec{E}}\sum_{q\in Q} u^q_{vw}\mu^q_{vw}$$

We can rewrite $(\mathbf{BIL})$ as the following linear integer program:

$(\mathbf{BILL})$:

$$\begin{aligned}
&\min_{y\in\{0,1\}^{|A|}} \sum_{(i,j)\in A} c_{ij}y_{ij}\\
&\text{s.t.} \quad \sum_{q=1}^{Q} y^q_e \le 1,\ \forall e\in E\setminus E_{\mathcal{I}}\\
&\qquad \sum_{q=1}^{Q}(y^q_{e_1}+y^q_{e_2}) \le 1,\ \forall (e_1,e_2)\in\mathcal{I}\\
&\qquad f(y)=0\\
&\qquad f(y) \ge \sum_{v\in V\setminus\{r\}} P_v\lambda_v + \sum_{(v,w)\in\vec{E}}\sum_{q\in Q} u^q_{vw}\mu^q_{vw}\\
&\qquad\qquad \forall b\in\mathcal{B}(y) \quad \forall \kappa,\lambda,\mu\in\mathcal{DX}(y,b) \qquad (26)\\
&\qquad y\in\{0,1\}^{|E|Q}
\end{aligned}$$

We use CPLEX as MIP-solver but there is an exponential number of constraints (26). To tackle this issue, we use a constraint generation algorithm (see for instance [1]): a relaxation of the formulation containing only a subset of Constraints (26) is first solved. Then a separation procedure is called. Let $\hat{y}$ be the current solution: we solve the linear program $f(\hat{y}) = \max_{b\in\mathcal{B}(\hat{y}),\kappa,\lambda,\mu\in\mathcal{DX}(\hat{y},b)} \sum_{v\in V\setminus\{r\}} P_v\lambda_v + \sum_{(v,w)\in\vec{E}}\sum_{q\in Q} u^q_{vw}\mu^q_{vw}$; let $\hat{b}$, $\hat{\kappa}$, $\hat{\lambda}$ and $\hat{\mu}$ be the solution. If $f(\hat{y}) > 0$, the constraint $\sum_{v\in V\setminus\{r\}} P_v\hat{\lambda}_v + \sum_{(v,w)\in\vec{E}}\sum_{q\in Q} u^q_{vw}\hat{\mu}^q_{vw} = 0$ is added to the set of constraints (26).

IV. Results analysis

In this section, we present the results of the tests for the formulations proposed for the design of wind farm cabling networks with load flow constraints. All experiments were performed on a computer with a 2.40GHz Intel(R) Core(TM) i7-5500U CPU and 16GB RAM, using the solver IBM ILOG CPLEX version 12.6.1, interfaced with Julia 0.6.0. We used in particular the package *JuMP*, a tool allowing mathematical modeling. For each test, the algorithm has been stopped after 3000 seconds if it has not terminated yet.

We introduce five real or subpart of real data sets $data_{10}$, $data_{23}$, $data_{28}$, $data_{35}$ and $data_{53}$ related to EDF offshore windfarms. Each data set $data_{|T|}$ contains a set of the $|T|$ wind turbines, their geographical location as well as the one of the sub-station. The graphs are partial grids with some diagonal edges. For each type of cable, we are given a cost per meter, a capacity and a susceptance. For the robust case, both formulations give about the same results and Table I presents the results of the tests.

The column I gives the instance on which the formulations are tested. The column $|Q|$ gives the number of types of cables that we consider for the instance. The column gap_f gives the final gap between the best integer solution found and the best lower bound (i.e. 0 if an optimal solution has been found). The column gap_r gives the gap between the best integer solution found and the best lower bound at the root node of the branch-and-cut. Finally, the column $time(s)$ gives the time to find the optimal solution (or 3000 if an optimal solution has not been found in 3000 seconds).

In Table I, the formulation for the non-robust case allows to solve exactly the problem for all instances with a number of wind turbines of at most 35, except for $data_{35}$ with $|Q|=3$ where it founds an integer solution within a gap of at most 0.04 to the optimal solution. For $data_{53}$, the final gap is 0.02 for $|Q|=1$, 0.06 for $|Q|=2$ and 0.14 for $|Q|=3$. The solving time or the final gap logically increase with the number of types of cables available, but the formulation still manages to find a solution within a reasonable gap from the optimum value.

For the robust case, the formulation is efficient especially for $|Q|=1$, where it solves all the instances to the optimum except $data_{53}$, for which the gap between the best integer solution and the optimal value is 8%, which sounds reasonable in this case. For $|Q|=2$, the formulation gets slower and is not efficient on $data_{53}$. However, it allows to solve optimally $data_{10}$ and $data_{28}$ and find an integer solution guaranteed to be within a really small gap of the optimum (0.3 %) for $data_{24}$. For $data_{35}$, the final gap is 10%. For $|Q|=3$, the formulation is not efficient on $data_{35}$ and $data_{53}$ but find an integer solution which is optimal or at least close to the optimal value (gaps of 4% and 2%) for the other data.

The robust formulation has a number of variables $|E|$ times bigger than the one of the non-robust one, which logically explains why it is importantly slower. Furthermore, each incrementation of $|Q|$ adds $3|E|$ variables for the non-robust formulation and $2|E||E|+|E|$ variables for the robust formulation. Logically, the incrementation of $|Q|$ has thus a higher impact on the robust formulation. Furthermore, the case where $|Q|=1$ corresponds to the case with uniform capacities, which appears to be easier to solve.

V. Conclusion

We presented an exact formulation to determine the best cabling of an offshore wind farm when no breakdown can occur. We have taken into account the load flow constraints

		Non-Robust			Robust		
I	$\|Q\|$	gap_f	gap_r	$time(s)$	gap_f	gap_r	$time(s)$
$data_{10}$	1	0	0.13	0.12	0	0	2.44
-	2	0	0.1	1.13	0	0.12	2.63
-	3	0	0.11	0.94	0	0.17	10.48
$data_{24}$	1	0	0.07	3.07	0	0.04	84.3
-	2	0	0.17	6.33	0.003	0.34	3000
-	3	0	0.14	8.21	0.04	0.41	3000
$data_{28}$	1	0	0.14	2.75	0	0.05	58.59
-	2	0	0.22	28.1	0	0.17	2349
-	3	0	0.26	77.9	0.02	0.29	3000
$data_{35}$	1	0	0.21	74.9	0	0.18	553
-	2	0	0.17	375	0.1	0.37	3000
-	3	0.04	0.33	3000	-	-	-
$data_{53}$	1	0.02	0.22	224	0.08	0.33	3000
-	2	0.06	0.2	3000	-	-	-
-	3	0.14	0.38	3000	-	-	-

TABLE I: Results of the tests

and shown how they can be formulated so as to be integrated into a mathematical model. Then we extended our model to find a robust network allowing any cable breakdown. We use a cutting plane method and IBM ILOG CPLEX to solve instances up to 53 wind turbines and obtain either an optimal solution or a good enough approximate solution. For instances of larger sizes, a heuristic approach will be required (see for instance [13]). The efficiency of this heuristic could be verified by comparing its results with the results obtained with our approach, on small instances.

For the robust case, we proposed two formulations which seem equivalent in case of one breakdown. Nevertheless, if we allow an arbitrary number of breakdowns, the number of variables and constraints becomes huge for the first linear formulation (Section III-A): with k breakdowns, we would have to consider $O(|V||E|^k + |E|Q)$ variables before linearization and we end up with an intractable model to solve. On the contrary, the bilevel formulation (Section III-B) is more compact and, most importantly, the number of variables doesn't get bigger with the size of the example. Thus, the bilevel formulation could be considered to search a robust networks in case of more than one breakdown.

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