# Open position for an Internship at *POEMS*

UMR 7231 CNRS/INRIA/ENSTA

2015-2016 Academic year

**Integral Equation methods for 2-D problems:**
**Study, implementation and validation of methods for singular integrals.**

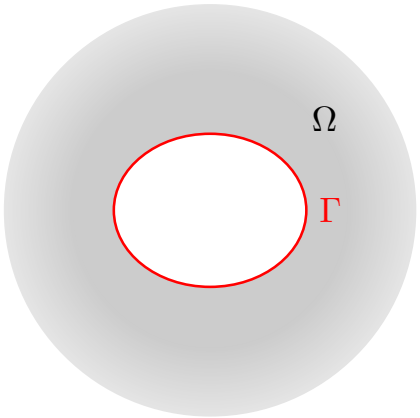
**Keywords**: wave problems, boundary element method, singular integrals, C/C++ programming.

▷ **Scientific context**

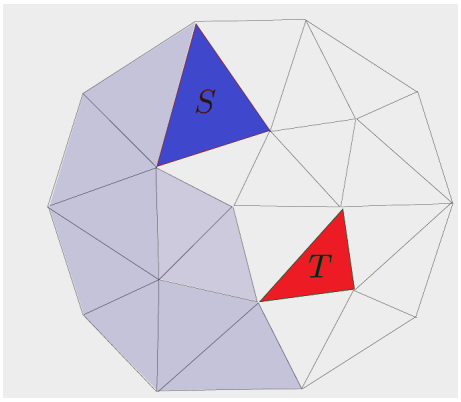
Boundary Integral Equations are well adapted to solve wave problems in infinite domain since they transform a problem defined in a domain $\Omega$ into an integral equation on the boundary $\Gamma = \partial\Omega$ of $\Omega$ (see Figure 1(a)). The numerical resolution of such integral equations can be done using the Boundary Element Method (BEM, see [1]). For the resolution, we introduce a mesh of $\Gamma$ (see Figure 1(b) for a 3-D problem). A difficulty of the BEM is that it requires the evaluation of integrals with singular integrands on the product of two elements of the mesh (for example triangles in 3-D or segments in 2-D). For the 2-D Laplace equation, one may have to evaluate the following integrals:

$$I = \int_{\alpha\times\beta} \psi(y)\phi(x)\log(\|x-y\|)dxdy, \text{ or } J = \int_{\alpha\times\beta} \psi(y)\frac{(x-y)}{\|x-y\|^2}\phi(x)dxdy \quad (1)$$

where $\alpha$ and $\beta$ are segments defining the boundary of the domain and $\psi$ and $\phi$ are basis functions.




(a) Domain $\Omega$ and its boundary $\Gamma$.




(b) Mesh of $\Gamma$ for 3-D problems.

The POEMS Team (Propagation des Ondes: Etudes Mathématiques et Simulation) at ENSTA-ParisTech is collaborating with IRMAR (Rennes) to develop a C++ code called XLiFE++ (eXtended Library of Finite Elements in C++) [5] for solving problems using Finite Element Method, Boundary Element Method and other methods.

We already have introduced methods for singular integrals arising in 3-D problems (see [2], [4] and [1]) but we have no methods in the code for singular integrals arising from 2-D problems.

▷ **Objectives**
The aim of this internship is to study and implement methods for the evaluation of integrals arising in 2-D problems in XLiFE++. The main objectives are:

1. Study methods to evaluate 2-D singular integrals in order to choose the most flexible method to implement.
2. Participation to the implementation of a method developed at ENSTA: the Method of Reduction of the Dimension [2, 3, 4] to compute explicitly several 2-D integrals.
3. Implementation and validation of these methods in the code XLiFE++.

The internship can be adapted depending on the person.

▷ **Knowledge.**
A background in Partial Differential Equations and specially in the Finite Element Method is expected in order to understand the basis of Boundary Element Methods. A basic knowledge of C/C++ is required.

▷ **Contact and location**
The internship will take place in the POEMS team at **ENSTA ParisTech** at Palaiseau in France, on the campus of the Ecole Polytechnique under the supervision of **Marc Lenoir** and **Nicolas Salles** and **Eric Lunéville**. The expected duration is **4-5 months**. Applications including CV and cover letter have to be sent by email.

The contacts are:


- **Marc Lenoir**
  Unité de Mathématiques Appliquées, ENSTA ParisTech, Palaiseau
  email: marc.lenoir@ensta-paristech.fr
- **Nicolas Salles**
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  email: nicolas.salles@ensta-paristech.fr


*The position is open to international students.*

# References


[1] Sauter, S. A., and Schwab, C. *Boundary element methods*. Springer, 2011

[2] Lenoir, M. *Influence coefficients for variational integral equations*. Comptes Rendus Mathematique, 2006.

[3] Lenoir, M., and Salles, N. *Evaluation of 3-D Singular and Nearly Singular Integrals in Galerkin BEM for Thin Layers*. SIAM Journal on Scientific Computing, 2012.

[4] Salles, N. *Calcul des singularités dans les méthodes d'équations intégrales variationnelles*. Thèse de doctorat de l'université Paris Sud, 2013.

[5] *XLiFE++ website :* http://uma.ensta-paristech.fr/soft/XLiFE++/.